Module 1: Texas Essential Knowledge and Skills (TEKS) Clarification: Grades 5-8

Module Introduction

This module focuses on clarifying the Texas Essential Knowledge and Skills in mathematics for grades 5-8.

In order to plan and teach a mathematics lesson that addresses a particular TEKS objective, it is very important to clarify the precise meaning of that TEKS. This clarification includes the mathematical content, the level of sophistication expected, necessary prerequisite knowledge and skills, and possible misconceptions or difficulties that students might have.

Objectives

This activity will provide you with a strategy for clarifying TEKS objectives so that your lessons will be as effective as possible. You will examine a grade 5 TEKS objective, clarify its meaning, compare it with other grade levels and with national standards, determine its prerequisites, and read research about student learning.

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Lesson 1: What is the Math Content?

Exploration

Let’s look at a TEKS for grade 5 mathematics:

5.2A Students will generate equivalent fractions.

Each of the terms in this TEKS has a meaning or interpretation that needs to be clarified. For each of the following terms, give (a) a definition and (b) an example of the term. You can use a glossary from a mathematics or methods textbook, a mathematics dictionary (e.g., wikipedia.com), or a Google search.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>Generate</td>
<td></td>
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<tr>
<td>Fraction</td>
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<tr>
<td>Equivalent fractions</td>
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**Follow-up Questions**

1. Why are 3/4 and 15/20 equivalent fractions?
2. What exactly would you expect to observe a student doing if he or she is generating equivalent fractions?
3. Write three fractions that are equivalent to 12/36. Are you generating equivalent fractions when you do this?
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Lesson 2: Level of Sophistication

The TEKS from Activity 1 could be interpreted in various ways, depending on the developmental level of the students or the experiences that students might have had. For example, some students may not be comfortable working with very large equivalent fractions; others may be challenged when working with symbols. According to the psychologist Vygotsky (as cited in Steele, 2001), optimal learning takes place when material is neither too difficult, nor too easy. This range of difficulty is called the zone of proximal development. Like Goldilocks’s porridge, it is just right.

Exploration

One way to determine the appropriate level of a TEKS is to look at other standards documents and compare their grade levels with the TEKS grade level. Important national documents include the National Council of Teachers of Mathematics (NCTM, 2000) Principles and Standards for School Mathematics (2000), NCTM Curriculum Focal Points (2006), and the American Association for the Advancement of Science (AAAS) Benchmarks for Science Literacy (1993).

In the following activity, make comparisons with TEKS 5.2A: Students will generate equivalent fractions.

1. Read the NCTM Number and Operations standard for grades 3-5.

   How do these expectations compare with TEKS 5.2A?

2. Read the NCTM Curriculum Focal Points for grade 5.
   http://nctm.org/standards/focalpoints.aspx

   How do these expectations at grade 5 compare with TEKS 5.2A?

3. Read the AAAS Computation and Estimation Benchmarks for grades 3-5 and 6-8.
   http://www.project2061.org/publications/bsl/online/ch12/ch12.htm#B

   How do the expectations at these grade levels compare with TEKS 5.2A?
4. Look at the TEKS 4.2 and 6.2 that relate to fractions.

http://www.tea.state.tx.us/rules/tac/chapter111/index.html

How are these TEKS different from 5.2A? How do these differences reflect levels of sophistication?

**Follow-up Questions**

1. Compared with the national standards expectations at this grade level, is TEKS 5.2A more sophisticated, less sophisticated, or about the same? Explain.

2. Which of these theories of learning do the TEKS in grades 3, 4, and 5 reflect: behaviorism, constructivism, or discovery? Explain.
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Lesson 3: Prerequisite Knowledge and Skills

It is important to know if students have the necessary prior knowledge and skills for a TEKS. Sometimes, difficulties in later grades can be traced back to missing knowledge or skill from very early grades. However, in this activity, we are most interested in the immediate prerequisites: things that should have been learned in a previous unit or perhaps from the previous grade.

Exploration

One way to identify prerequisites is to do an activity that addresses the TEKS, keeping a list of all the mathematical ideas and skills that you apply as you think about and complete the activity. That is what you will do next. For each step, record the mathematical knowledge or skills you use to complete it.

<table>
<thead>
<tr>
<th>Step</th>
<th>Math knowledge or skills used</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Write the fraction three-eighths</td>
<td>$\frac{3}{8}$ = $\frac{3\times1}{8}$</td>
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<tr>
<td>b. Write $\frac{3}{8}$ = $\frac{3\times2}{8}$</td>
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<tr>
<td>c. Write $\frac{3}{8}$ = $\frac{8}{8\times2}$</td>
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<tr>
<td>d. Write $\frac{3}{8}$ = $\frac{6}{16}$</td>
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<tr>
<td>e. Explain why $\frac{3}{8}$ and $\frac{6}{16}$ are equivalent fractions.</td>
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Follow-up Questions

1. Which of these knowledge and skills are contained in other 5th grade TEKS?
2. Which of these knowledge and skills are contained in TEKS from earlier grades?
3. What concrete model could be used to generate equivalent fractions? Would the prerequisites be different if concrete models were used?
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Lesson 4: Student Misconceptions

Part of planning and teaching a lesson involves determining whether your students have misconceptions about the content. A misconception happens when a student believes something that is false, or partially true. For example, some students believe that when a number is multiplied by another number, the result will always be larger than the original number. This belief is very problematic when they begin working with fractions. These misconceptions may have been developed from previous learning or when there was an over-emphasis on skill development without conceptual understanding (Hiebert & Wearne, 1986). Misconceptions are often difficult to change, especially when they give the correct answer some of the time.

There is a difference between errors and misconceptions. An error or mistake happens when a fact or concept is not recalled correctly or when a “slip” is made in writing a step in a procedure or an answer. Mistakes are usually made somewhat at random and do not represent an underlying lack of conceptual understanding. Some errors do occur in patterns, and these may mean there is a misconception, or that the student has not learned the procedure completely. For example, students sometimes add fractions as shown below:

\[
\frac{2}{3} + \frac{3}{5} = \frac{2+3}{3+5} = \frac{5}{8}
\]

Exploration

Read the following research summary from the Resources section of this gradeband, discussing students’ understanding of fractions. As you read it, keep in mind the prerequisite skills and knowledge for TEKS 5.2A, especially those that are needed to explain why fractions are equivalent.

Research from Benchmarks for Science Literacy

Upper-elementary- and middle-school students may exhibit limited understanding of the meaning of fractional number (Kieren, 1992). For example, many seventh-graders do not recognize that 5 1/4 is the same as 5 + 1/4 (Kouba et al., 1988). In addition, elementary-school students may have difficulties perceiving a fraction as a single quantity (Sowder, 1988) but rather see it as a pair of whole numbers. An intuitive basis for developing the concept of fractional number is provided by partitioning (Kieren, 1992) and by seeing fractions as multiples of basic units—for example, 3/4 is 1/4 and 1/4 and 1/4 rather than 3 of 4 parts (Behr et al., 1983).
Elementary- and middle-school students make several errors when they operate on decimals and fractions (Benander & Clement, 1985; Kouba et al., 1988; Peck & Jencks, 1981; Wearne & Hiebert, 1988). These errors are due in part to the fact that students lack essential concepts about decimals and fractions and have memorized procedures that they apply incorrectly. Interventions to improve concept knowledge can lead to increased ability by fifth-graders to add and subtract decimals correctly (Wearne & Hiebert, 1988).

Students of all ages misunderstand multiplication and division (Bell et al., 1984; Graeber & Tirosh, 1988; Greer, 1992). Commonly held misconceptions include “multiplication always makes larger,” “division always makes smaller,” and “the divisor must always be smaller than the dividend.” Students may correctly select multiplication as the operation needed to calculate the cost of gasoline when the amount and unit cost are integers, and then select division for the same problem when the amount and unit cost are decimal numbers (Bell et al., 1981). Numerous suggestions have been made to improve student concepts of multiplication (Greer, 1992), but further research is needed to determine how effective these suggestions will be in the classroom.

Lower-middle-school students may have difficulties understanding the relationship between fractions and decimal numbers (Markovits & Sowder, 1991). They may think that fractions and decimals can occur together in a single expression, like 0.5 + 1/2, or they might believe that they must not change from one representation to the other (from 1/2 to 0.5 and back) within a given problem. Instruction that focuses on the meaning of fractions and decimals forms a basis on which to build a good understanding of the relationship between fractions and decimals. Instruction that merely shows how to translate between the two forms does not provide a conceptual base for understanding the relationship (Markovits & Sowder, 1991).

**Follow-up Questions**

1. Which research finding was most surprising to you? Why?
2. How would the idea of partitioning—or seeing fractions as multiples of basic units (for example, 3/4 is 1/4 and 1/4 and 1/4)—help students understand the process generating fractions by using the property \( a \times 1 = a \)?
3. Suppose a student generated fractions equivalent to 2/3 like this:

\[
\frac{2}{3} = \frac{2+2}{3+3} = \frac{4}{6} = \frac{4+2}{6+3} = \frac{6}{9}
\]

Is this method correct? Explain. How would you respond to this student?
References for Further Reading


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Assessment

1. Why are 2/3 and 10/15 equivalent fractions?
2. A student is asked to simplify the fraction 12/36. Is he/she being asked to find an equivalent fraction? Explain.
3. A student is asked to explain how to find two fractions equivalent to 3/5. He says: “double the 3 and 5 and you get 6/10; then double the 6 and the 10 and you get 12/20.” Does this student understand generating equivalent fractions? Explain.
4. Suppose a particular 6th grade TEKS is more sophisticated, compared with an NCTM standard. What could make it more sophisticated?
5. Suppose a particular 6th grade TEKS is contained in a grade 3-5 AAAS benchmark. Is the TEKS more or less sophisticated than the benchmark?
6. A teacher decides to use coins as a concrete model to generate equivalent fractions. What would be a disadvantage to this model?
7. Would you call the student’s work in #3 a misconception? Explain.
8. Suppose a student generates fractions equivalent to 2/3 like this:

\[
\frac{2}{3} \times \frac{10}{10} = \frac{20}{30} \quad \frac{2}{3} \times \frac{100}{100} = \frac{200}{300}
\]

What question would you ask the student to determine if he/she understands how to generate equivalent fractions?

This concludes the TEKS Clarification Module. To continue to the FDP Equivalency Module, click on the right arrow button below.