Module 4: Student Thinking

Misconceptions in Place Value and Fractions

Module Introduction

This module focuses on the concept of student thinking.

Before you begin this module, read the following article by Allen (2007) on misconceptions to give you some background information.

*What is a Misconception?*

*Want to Know Questions*

After reading this article, write five questions you would like to be able to answer at the end of this module.

Several researchers (Baroody 1989; Fuson 1990; Kamii 1989) have identified and discussed student misconceptions of place value, noting students with experiences in place-value learning are often unable to explain the meaning of "places" of numerals. Place value is the key to teaching computation in our base-ten numeration system (Sowder 1997) but is more than just identifying place values. Students should be able to explain and model what each "place" represents. Do students understand a "ten" represents a single grouping of ten "ones"? Students "… are often able to disguise their lack of understanding of place value by following directions, using the tens and ones in prescribed ways, and using the language of place value" (Van de Walle 2007 p. 200). Wearne and Hiebert (1994) stated "understanding place value involves building connections between key ideas of place value--such as quantifying sets of objects by grouping by ten and treating the groups as units--and using the structure of the written notation to capture this information about grouping" (p. 273). Students having difficulty with computation often have not learned to combine the concepts of place value and face value (Ashlock 1994) – the product value of a number is the product of a digit's face value and its place value, e.g., 28 is \((2 \times 10) + 8 \times 1\).

Student misconceptions may be exposed through discussions and explanations. Student explanations may reveal either a total lack of understanding or perceptions contradicting accepted mathematical practice (Campbell 1997). Many misconceptions and faulty thinking in arithmetic, e.g., fractions, result in misconceptions and faulty thinking in algebra. "Without a solid understanding of place value, children reach a roadblock in mathematics because they lack the skills and knowledge needed for higher-level thinking" (Nagel & Swingen 1998 p. 164).

Comparing the size of fractions by considering only the size of the denominator can also be considered a case of regarding the numerator and denominator as two unrelated whole
numbers. This misconception is prevalent. The tendency to choose as the larger fraction the one with the larger denominator has also been reported in the literature (e.g. Baroody & Hume, 1991).

There are many factors that may contribute towards elementary school students’ poor understanding of common fractions. Based on the research results reported by, for example, Baroody & Hume (1991); Streefland (1991) and D’Ambrosio & Mewborn (1994), as well as projects e.g. Murray et al. (1996), there appear to be three main possible causes:

1. The way and sequence in which the content is initially presented to the students, in particular exposure to a limited variety of fractions (only halves and quarters), and the use of pre-partitioned manipulatives;
2. A classroom environment in which, through lack of opportunity, incorrect intuitions and informal (everyday) conceptions of fractions are not monitored or resolved; and
3. The inappropriate application of whole-number schemes, based on the interpretation of the digits of a fraction at face value or seeing the numerator and denominator as separate whole numbers.

Suppose you have two pizzas of the same size. You cut one of them into six equal-sized pieces and you cut the other one into eight equal-sized pieces. If you get one piece from each pizza, which one do you get more from?' All students responded that they would get more from the pizza cut into six pieces... Each student also was asked a question like 'Tell me which fraction is bigger, 1/6 or 1/8.'... Four of the students who first solved the real-world problem and three students who first worked with the symbolic representation responded, 'One eighth is bigger" (Mack, 1990, p. 22).

Some guidelines for guiding instruction for students having difficulty learning to compute are as follows (Ashlock, 1994):

- Personalize instruction
- Believe the student is capable of learning
- Make sure the student knows what is needed
- Encourage self-assessment
- Ensure consistent expectations
- Provide feedback on progress
- Build on actual prior knowledge of the student
- Emphasize ways to organize what is learned
- Stress estimation
- Base instruction on diagnosis
- Use a variety of instructional strategies and activities
- Involve student in higher-order thinking activities
- Connect content to real-life
- Encourage think-aloud while working
- Use questioning strategies to encourage reflection
• Encourage student to explain understanding of a concept in his or her own words
• Sequence instruction in smaller blocks of content
• Move to symbols gradually
• Emphasize careful penmanship and proper alignment of digits
• Allow student choice of materials if possible
• Encourage use of hands-on materials as long as of value
• Assure student understanding of process before assigning practice
• Use activities which provide immediate feedback
• Use short practice sessions.

Reflection Questions:

1. How does understanding or identifying student misconceptions enhance your lesson planning and successful classroom instruction?
2. What are five guidelines which will be most useful to you in working with students’ misconceptions?
Module 4: Student Thinking

Lesson 1. Student Thinking and Place Value

Whole Numbers: Addition and Subtraction

Objectives

At the completion of this module the participant will be able to

- identify error patterns students make in computation,
- verify student error patterns by using the pattern in sample problems, and
- determine appropriate strategies to implement in correcting errors involving place value concepts and computation.

To plan effective instruction, we should be aware of the knowledge of students and the nature of any incorrect or mis-constructed computations. We should be aware of error patterns in student work.

Readings and Activities

These activities are adapted from Ashlock’s work 9th edition.

In the following examples, you will find student work in adding and subtracting whole numbers. These examples, containing the error patterns of actual students observed by teachers in ordinary school settings, provide you with the opportunity to develop your own skill in identifying error patterns.

In the examples look for a pattern of errors and then check your observation by using the error pattern yourself. Be careful not to make a quick assessment. (Students often make quick decisions, thus, using and constructing the type of incorrect procedures you see.) When you decide you have determined a pattern, check your hypothesis by examining the other examples of the student's work.

Also, as you read about and examine each student’s procedure, think about the instruction strategies needed for Gloria, Juanita, Keisha, and Joe. Students need to experience multiple ways of exploring problems. Try to determine at least two activities to help the student.
Error Pattern for Gloria

Gloria gets some correct sums, but many of her results are unreasonable. What error pattern is Gloria following in her written work?

Check to see if you found Gloria’s pattern by using her incorrect procedure to compute these examples.

Why would Gloria use such a procedure? Is she estimating? What does this tell you about her number sense?

Planning Instruction for Gloria

Gloria’s pattern is a reversal of the procedure used in the usual algorithm without regard for place value. Gloria adds from left to right, and when the sum of a column is 10 or greater, she records the left figure and places the right figure above the next column to the right.

If you were Gloria’s teacher, what corrective procedures could you implement? One strategy could be as follows:

1. Use a game-like activity with a pattern board, base blocks, and a bank. By making the algorithm a written record of moves in a game-like activity, you can help students understand place values and begin computation with units. A pattern board serves as an organizing center.
2. Show how you would use base ten blocks and show the sum.
**Error pattern for Juanita**

There were 62 students in Ms. Jones’s gym class. Nine more students were added to the class. How many students are in the class now? When Juanita adds, she gets many of her answers correct, but she came up with 17 students for this problem. That is not a reasonable answer. What error pattern did she utilize?

<table>
<thead>
<tr>
<th>Name</th>
<th>Juanita</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 48</td>
<td>B. 17</td>
</tr>
<tr>
<td>+ 3</td>
<td>+ 20</td>
</tr>
<tr>
<td>15</td>
<td>37</td>
</tr>
<tr>
<td>C. 7</td>
<td>D. 36</td>
</tr>
<tr>
<td>+ 23</td>
<td>+ 23</td>
</tr>
<tr>
<td>12</td>
<td>59</td>
</tr>
<tr>
<td>E. 75</td>
<td>+ 8</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

Did you find her error pattern? Check yourself by using her procedure to determine these sums.

| F. 32 | G. 50 |
| + 4   | + 38  |
| H. 35 |       |
| + 2   |       |

What does Juanita understand about numerals and estimating? Do her answers make sense? Why would she use such a procedure?

**Teaching Strategies for Juanita**

Juanita misses problems in which one of the addends is written as a single digit. She adds the three digits as if they were all units. (When both addends are two-digit numbers, she seems to add correctly. However, Juanita may not be applying any knowledge of place value with either type of example. She may be just adding units in straight columns. When one addend is a one-digit number, she adds the three digits along a curve.) If Juanita is adding in this method, she may have even more failure and frustration when she begins adding and subtracting involving renaming.

1. Describe how an interview with Juanita may provide helpful information. Is she able to explain the examples that were worked correctly? Does she identify tens and units? Does she explain that units must be added to units, and tens must be added to tens?
2. How would you help Juanita? Demonstrate how you could draw a line to separate tens and units.
3. Show one other strategy you could use to help Juanita the next time she must add.
Error Pattern for Keisha

Keisha’s answers seem reasonable to her, and most of them are correct. But in subtraction she keeps getting incorrect answers. See if you can find her pattern in her written work.

Do you find an error pattern? Check your idea by following her procedure for these examples.

Why would a student use such a procedure? Has Keisha learned something that she is using inappropriately?

Teaching Strategies for Keisha

Although Keisha subtracts 0 – 0 = 0 correctly, she writes “0” for the missing addend (difference) whenever the known addend (subtrahend) is zero. She may be confusing this situation with multiplication combinations in which zero is a factor. She writes nine of the basic subtraction combinations incorrectly because of this one difficulty. We should be able to help Keisha with this type of error.

1. How would you help her using base blocks or bundled sticks to picture the computation?
2. Show one more strategy to help Keisha with subtracting.

Error Pattern for Joe

Joe recently learned to regroup (or “rename”), and initially he had correct answers. But then there were difficulties when he encountered simple problems such as the following:
Lou had 168 pieces in his set of plastic blocks. But he traded 54 of them for some football cards. How many pieces are in his set of plastic blocks now?

Look carefully at Joe’s written work. Is he using estimation?

What does he understand about numeration and subtraction? What does he not understand? What error pattern did he use? Check yourself by using his procedure for these examples.

Why would someone use such a procedure?

**Teaching Strategies for Joe**

Joe has learned to “borrow,” or rename, in subtraction. He renames whenever he subtracts. Joe could possibly rename one ten as 10 ones and possibly interpret the answer as 1 hundred, 1 ten, and 11 ones. His final answer does not use conventional place-value notation.

Joe’s procedure is correct for a subtraction example requiring renaming tens as ones but he has over generalized. He does not differentiate between examples requiring renaming and those not requiring renaming. Since some of his answers are correct, he may think the procedure he is using is appropriate for all subtraction examples.

1. How would you help Joe replace his error pattern with a correct computational procedure?

Note: Useful instruction will emphasize (1) the ability to distinguish between subtraction problems requiring regrouping in order to use basic subtraction combinations and subtraction problems not requiring regrouping, and (2) the mechanics of notation.
Follow-up Questions

Be prepared to discuss the following questions:

1. What should you focus on as you work with students to prevent such errors in computational procedures?
2. How does terminology impact the development of student errors?
Module 4: Student Thinking

Lesson 2. Student Thinking and Fractions

MISCONCEPTIONS—Fractions: Concepts and Equivalence

Lesson Introduction

When planning instruction, it is important to look at examples of student work in an effort to identify erroneous or incomplete understanding of mathematics concepts. Teachers who are aware of errors in their students’ work are better able to plan lessons that address student needs.

Objectives

At the completion of this module the participant will be able to

- identify error patterns students make in working with fractions,
- verify student error patterns by using the pattern in sample problems, and
- determine appropriate strategies to implement in correcting errors involving fraction concepts.

Standards Addressed

TEKS: State Mathematics Standards: TEKS on Fractions

NCTM Standards:

In prekindergarten through grade 2 all students should –

- understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2.

In grades 3-5 all students should –

- develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;
- use models, benchmarks, and equivalent forms to judge the size of fractions; and
- recognize and generate equivalent forms of commonly used fractions, decimals, and percents.

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**Readings and Activities**

This lesson involves students' understanding of fractional concepts and equivalence. Students used incorrect procedures in attempting to find the correct answers. Look for the misconceptions that are located in each of the four students' answers that follow. Try to locate the error patterns that each of the students have made. Look carefully and check each example to make sure that you have found the pattern.

Be ready to plan some strategies to use with each of the students to help correct their error patterns. Develop more than one strategy for Grace, Jose, Carol, and Sharon since it will usually require a variety of teaching strategies to overcome the error pattern that a particular student has developed.

**IDENTIFYING PATTERNS**

**Error Pattern for Grace**

Grace found this worksheet easy, but she only got one of the problems correct. Ask yourself, what does she understand and what does she not understand? She is demonstrating an error pattern–can you figure out what it is?

![Image of worksheet](image-url)
What made Grace able to correctly answer Example B? Look through all of the problems she completed to assure that you have located her error pattern.

How can you assist Grace so that she will demonstrate accurate symbolic representations for fractions?

![Image of fractions](image)

**PLANNING INSTRUCTION FOR GRACE**

Obviously Grace knows that the number of shaded parts is shown by the numerator. She counts the number of parts that are not shaded and writes that number as the denominator of the fraction. She comprehends that fractions are part to part, rather than part to whole.

Grace’s only correct response was Example B. Possibly she memorized that a semi-circular shape is one half; she might think that a semi-circular region may be “what one half looks like.”

Can you assist Grace in correcting her error pattern? One strategy is given.

Grace needs to find a way to show that a fractional part of a quantity is connected to the unit whole.

1. Begin with the whole. She should make the fraction for a specific unit region by thinking about the unit whole. How many equal pieces are there in the whole rectangle? Have her write that number. Next she should put a line over that number and write the number of shaded parts over the line. Grace must read every fraction she makes (reading from the top to the bottom): one of three parts, is one third; two of four parts, is two fourths. If she was working with a set, she should use colored pencils or crayons to demonstrate the fractional part of the set.

2. How could you use fraction strips to help Grace?

**Error Pattern for Jose**

Jose must find the simplest terms for each fraction. He is presented with some problems that are already simplified. Some fractions must be simplified.
What process is he using to simplify the problems? Finding this error pattern might difficult. Sometimes it may be necessary to talk and listen to students so you can understand their thinking.

Although he was demonstrating an error pattern, Jose changed some fractions to simplest terms. See if you found Jose’s procedure by using his error pattern to answer problems e and f.

How could you assist Jose in correcting this error pattern?
Planning instruction for Jose

Some of Jose’s answers are actually correct. Apparently he does not know which fractions are in their simplest terms. After interviewing Jose, you determine that his explanation is:

“3 goes to 1, and 4 goes to 2”
“3 goes to 1, and 9 goes to 3”
“3 goes to 1, and 8 goes to 4”
“4 goes to 2, and 8 goes to 4”

Jose connects a specific whole number with every numerator or denominator. All 3s become 1s and all 8s become 4s when fractions are simplified. Jose did not need to understand fractional equivalence to use this mechanical process. Sometimes this allows him to obtain a correct answer.

Students have great difficulty developing an understanding of the concept of equivalent fractions, and this takes time and effort. Jose has great difficulties. Try to help him replace his error pattern with a correct procedure. Find 2 strategies you could use: three suggestions are provided below or describe your own strategy.

1. Illustrate how you could have Jose utilize fractional parts of areas.
2. Explain how you could assist Jose by having him build an array with fractional parts of a set.
3. Describe how you could have Jose locate patterns.

Error Pattern for Carol

Carol completed some tasks demonstrating an understanding of some fractional concepts. Does she possess a total understanding of fractions? What does she need to understand? Try to locate her error pattern.
Are you certain that you know what Carol is doing by using her process with these examples?

Planning Instruction for Carol
Carol appropriately links the denominator of the fraction with the total number of parts shown and the numerator with the amount of darkened parts.

Obviously Carol doesn’t understand that these associations pertain only when all the darkened sections are equivalent.

Every fractional part must be the same part of the whole unit, covering the same part within the unit whole.

She must make sure that all parts are equivalent. If they are not equivalent, she needs to verify how much of the unit whole each darkened part covers and write a fraction.

1. If you were the tutor in the classroom, how could you help Carol eliminate her error pattern? Illustrate two strategies you could use to assist Carol.

Error Pattern for Sharon
Sharon understands fundamental fractional concepts and what equals means, but she undoubtedly is not demonstrating that understanding. She seems to be using an idiosyncratic rule that she has made up, perhaps from something she has mistakenly understood from her teacher or other students.

She attempted to simplify all the fractions to lowest terms, but Sharon’s answers do not make sense. Try to locate
Sharon’s error pattern:

| a) \(\frac{4}{12} = \frac{3}{12}\) | b) \(\frac{6}{10} = \frac{1}{10}\) | c) \(\frac{2}{4} = \frac{2}{4}\) |
| d) \(\frac{9}{9} = \frac{1}{9}\) | e) \(\frac{4}{6} = \frac{1}{6}\) | f) \(\frac{12}{3} = \frac{3}{12}\) |

Apply her method with these fractions to determine if you located her pattern.

| g) \(\frac{3}{6} = \) | h) \(\frac{8}{3} = \) |

**Planning Instruction for Sharon**

Sharon looks at the particular numerator and denominator as two whole numbers and divides the larger by the smaller to get the new numerator. In addition, she pays no attention to any remainder; then she writes the greater of the two numbers as the new denominator. Maybe Sharon has seen in most places that the denominator is the larger of the two numbers.
Sharon is only moving symbols in a rote manner and she is not understanding fractions as part of unit wholes. Evidence of this is shown when she writes that $6/10 = 1/10$, suggesting that an understanding of $3/6$ or $2/6$ as parts of a unit just is not present. Or, if it is present, it is a behavior associated with something like fraction pies and is not applied in other contexts. Also, she probably thinks of equals as “the answer comes next” instead of “is the same as.”

Before deciding upon corrective tutoring, it may be necessary for the teacher to conference with her to find out exactly what she understands regarding fractions and equals.

Similar to other students, Sharon uses rote and irrational actions with mathematical tasks. How can you assist her? Explain two strategies you could employ to help her eliminate her error pattern.

**CONCLUSION**

A variety of manipulatives and diagrams are available for modeling fractional quantities. Equivalent parts of unit wholes, fraction strips and circles, and number lines are some choices. Assortments of models are available and are necessary so that students do not always connect one model with a certain fractional quantity.

As you assist each student, learn procedures for changing a fraction to equivalent fractions, and focus on concepts and number sense. Make sure that students get used to asking themselves if their answers make sense. Students should ask if both numbers are less than a half. A number line can help them see if the number is close to one. Educated guesses and sensibleness of answers should consistently be in the minds of students.

Don’t forget that diagnosis continues even during instructional activities, as you examine students working on a task. Continue observing to find error patterns so that you can help students eliminate them.

**Follow-up Activity**

Be prepared to discuss the following questions in class.

1. How can you identify student errors in understanding fraction concepts?

2. How can the use of manipulatives affect student misconceptions?

3. How will students’ misconceptions/error patterns with fractions affect their understanding of mathematical concepts in the middle grades? Give examples.
Extensions

   - Click on Workshop #6: Ratio and Proportion: When a Third is More Than a Half.
2. Discuss the concepts the presenters think are important to help students gain a correct understanding of fractions.
3. What misconceptions are presented in the video?
4. How will students’ misconceptions of fractions in the early grades affect their learning mathematical concepts in the middle grades?
Checking for Understanding- Module 4

1. A ______________ is a mistaken idea or view resulting from a misunderstanding of something.

2. Misconceptions are but one facet of ______________, ________________, or ______________ thinking.

3. Misconceptions definitely ______________ with learning when students use them to interpret new ideas and concepts.

4. ______________ ______________ refers to that which is taught in an organized, structured, educational institution such as a system of interrelated definitions and proofs.

5. ______________ ______________ is generated or learned through one’s personal actions; that is, it refers to routines that are carried out mechanically or by habit.

6. Student misconceptions may be exposed through ______________ and ______________, revealing either a complete lack of ______________ or perceptions ______________ accepted mathematical practice.

7. The best way to dispel a misconception is for the student to discover a ______________.

8. Student misconceptions of ______________ ______________ have been identified as students are often unable to explain the meaning of places of numerals.

9. Various misconceptions and flawed thinking in arithmetic result in misconceptions and incorrect thinking in ______________.

This concludes the Student Thinking Module. You may select the previous button below to go back or choose a new option from the menu at the left of this page.